Nontrivial, Asymptotically Non-free Gauge Theories

and
Dynamical Unification of Couplings

JISUKE KUBO [†]

College of Liberal Arts, Kanazawa University, Kanazawa 920-11, Japan

Abstract

An evidence for nontriviality of asymptotically non-free (ANF) Yang-Mills theories is found on the basis of optimized perturbation theory. It is argued that these theories with matter couplings can be made nontrivial by means of the reduction of couplings, leading to the idea of dynamical unification of couplings (DUC) The second-order reduction of couplings in the ANF $SU(3)$-gauged Higgs-Yukawa theory, which is assumed to be nontrivial here, is carried out to motivate independent investigations on its nontriviality and DUC.

[†]E-mail address: jik@hep.s.kanazawa-u.ac.jp

Asymptotically free (AF) theories [1] do not suffer from the problem of triviality [2]. It is widely believed that the question of triviality cannot be addressed within the framework of perturbation theory, and so far there is no real indication for the existence of a nontrivial four-dimensional theory that is not AF. It is however tempting to think that if an infrared-free theory has an ultraviolet fixed point which is small, perturbation theory might be intact even near the fixed point and hence could be applicable to the triviality problem.

About ten years ago, Sakakibara, Stevenson, and I [3] considered perturbation theory near a fixed point. We formulated the problem as the problem of the renormalization scheme (RS) dependence, because at any finite order in perturbation theory even the existence of a positive zero of the β-function depends on RS. We performed our investigation on the basis of optimized perturbation theory (OPT) [4], which as well known yields RS-invariant perturbative approximations and has already experienced certain successes in perturbative QCD [5] and also in QED [6]. We found that one needs a perturbative calculation of a physical quantity of at least third order in order to be able to apply our method. Investigating concrete field theory examples, we argued that under certain circumstances perturbative analyses based on OPT near a fixed point could be believable.

Recently, using the third-order QCD corrections of the $e^+ e^-$ cross sections [7], Mattingly and Stevenson [8] applied OPT and concluded that AF QCD has an infrared-stable fixed point. Although the assumption on the existence of an infrared fixed point in AF QCD has no logical inconsistency, it is not clear at all how much the fixed point found in a perturbative approach can describe the physics in the infrared regime, because the nonperturbative effects play the essential role in understanding the low energy physics of AF QCD. In the ultraviolet regime, on the other hand, the nonperturbative nature may be neglected in describing the basic part of the physics of AF QCD.

One of the main assumptions of this note, which is partly motivated by this fact, is that this is true even in asymptotically non-free (ANF) Yang-Mills theories. Of course, this is a very strong assumption, but there is neither internal inconsistency of this assumption, nor known fact against it (at least to my knowledge [9]). Moreover, as it will be seen, the investigation based on OPT indicates that ANF Yang-Mills theories could

have an ultraviolet fixed point so that they could be well-defined, interacting theories in the ultraviolet limit.

We begin by recalling the basic result obtained in Ref. [3]. Consider a physical quantity $\mathcal{R}(p_k, \mu, \alpha(\mu)/\pi)$ in a massless renormalizable theory, where p_k stand for the physical external momenta, μ is the renormalization scale, and $\alpha(\mu)$ is the renormalized coupling. In the nth order of perturbation theory, $\mathcal{R}$ can be written as $\mathcal{R}^{(n)}(p_k, \mu, a) = \gamma a [1 + \sum_{i=1}^{n-1} r_i(p_k, \mu) a^i]$, $a \equiv \alpha/\pi$, while the β-function takes the form $\beta^{(n)}(a) = -a^2 \sum_{i=0}^{n-1} b_i a^i$ The coefficients b_i's ($i \geq 2$) are RS-dependent, and along with μ they can uniquely parameterize the RS-dependence. Therefore, $\mathcal{R}$, being a physical quantity, has to satisfy $\mu \partial \mathcal{R}/\partial \mu + \beta \partial \mathcal{R}/\partial a = 0$ and also $\partial \mathcal{R}/\partial b_i = 0$ ($i \geq 2$), which we altogether symbolically denote by $d\mathcal{R}/d(\text{RS}) = 0$. Then the essence of OPT is to demand the optimization condition [4]

$$\frac{d\,\mathcal{R}^{(n)}}{d\,(\text{RS})}\bigg|_{\text{RS}=\text{optRS}} = 0 \ , \tag{1}$$

and to fix from this an optimized RS for a given physical quantity. Note that in perturbation theory one has only $d\mathcal{R}^{(n)}/d(\text{RS}) = O(a^{n+1})$. In Ref. [3], we assumed that OPT makes sense even near a fixed point and found that the fixed point a^*_{opt} in the third order can be obtained from

$$0 = \frac{7}{4}\frac{b_0}{b_1} + a^*_{\text{opt}} + 3\frac{b_0}{b_1}\rho_2\,(a^*_{\text{opt}})^2 \ , \tag{2}$$

$$\rho_2 = r_2 + \frac{b_2}{b_0} - (\,r_1 + \frac{1}{2}\frac{b_1}{b_0}\,)^2 \ , \tag{3}$$

where ρ_2 is the RS-independent quantity for a given $\mathcal{R}$. From Eq. (2), one sees that the more negative the ρ's are, the more likely is the existence of a positive a^*_{opt}.

What follows is a slight generalization of the analysis of Ref. [8] in QCD, but with completely different physics and its applications in mind. The β-function coefficients of the first three orders in the $\overline{\text{MS}}$ scheme can be found in Refs. [10]:

$$b_0 = \frac{11}{6}C_A - \frac{2}{3}T_F f \ , \ b_1 = \frac{17}{12}C_A^2 - (\,\frac{5}{6}C_A + \frac{1}{2}C_F\,)\,T_F f \ , \tag{4}$$

$$b_2 = \frac{2857}{1728}C_A^3 + (\,-\frac{1415}{864}C_A^2 + \frac{79}{432}C_A T_F f - \frac{205}{288}C_A C_F$$
$$+ \frac{11}{72}C_F T f + \frac{1}{16}C_F^2\,)\,T_F f \ , \tag{5}$$

where C_A, C_F and T_F are the usual group theoretic coefficients. ($C_A = N, C_F = (N^2 - 1)/2N, T_F = 1/2$ for the $SU(N)$ gauge theory with the Dirac fermions in the fundamental representation.) Asymptotic non-freedom requires that $f > 11C_A/2$, and I concentrate only on such cases from the reason given before. I will below calculate ρ_2 in the ANF $SU(2)$ and $SU(3)$ gauge theories with f Dirac fermions in the fundamental representation. To this end, I use the third-order corrections to (A) $\sigma_{\text{tot}}(e^+e^- \to \text{hadrons})$ [7] and (B) the Gross-Llewellyn Smith sum rule for deep inelastic neutrino-nucleon scattering [11].

A. $\sigma_{\text{tot}}(e^+e^- \to \text{hadrons})$

The first quantity is the so-called R-ratio $R(s/\mu^2, a(\mu)) = d_R \sum_f Q_f^2 (1 + \mathcal{R}(s/\mu^2, a))$, which is defined by $\sigma_{\text{tot}}(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ in the e^+e^- annihilation, where s is the center of mass energy, d_R is the dimension of the quark representation, and Q_f stands for the electric charge of the "f" quark. Since it is unlikely that the real electric charge of the quark is related to the existence of a fixed point in a nonabelian gauge theory, I instead use the fermion number and assume that $Q_f = 1$ for all the fermions. Under this assumption, I recall the third-order result of Ref. [7]:

$$\mathcal{R}^{(3)} = \frac{3}{4}C_F a (1 + r_1 a + r_2 a^2),$$ (6)

where

$$r_1(s/\mu^2 = 1) = [\frac{41}{8} - \frac{11}{3}\zeta(3)]C_A - \frac{1}{8}C_F + [-\frac{11}{6} + \frac{4}{3}\zeta(3)]T_F f,$$ (7)

$$\begin{aligned}
r_2(s/\mu^2 = 1) &= [\frac{90445}{2592} - \frac{2737}{108}\zeta(3) - \frac{121}{432}\pi^2]C_A^2 - [\frac{127}{48} + \frac{143}{12}\zeta(3)]C_A C_F - \frac{23}{32}C_F^2 \\
&+ 55[-\frac{1}{18}C_A + \frac{1}{3}C_F]\zeta(5)C_A + [\frac{302}{81} - \frac{76}{27}\zeta(3) - \frac{1}{27}\pi^2]T_F^2 f^2 \\
&+ [\frac{11}{144} - \frac{1}{6}\zeta(3)]\frac{d^{abc}d^{abc}}{C_F d_R}f + [(-\frac{1940}{81} + \frac{448}{27}\zeta(3) + \frac{10}{9}\zeta(5) + \frac{11}{54}\pi^2)C_A \\
&+ (-\frac{29}{48} + \frac{19}{3}\zeta(3) - \frac{20}{3}\zeta(5))C_F]T_F f.
\end{aligned}$$ (8)

Using these three- and four-loop results, one can now computes ρ_2 defined in Eq. (3):

$$\begin{aligned}
\rho_2 &\simeq [-4.2140 + 0.03224f + 0.05455f^2 - 8.12 \times 10^{-4}f^3 \\
&\quad -1.53 \times 10^{-4}f^4] \cdot [1 - f/11]^{-2} \quad \text{for} \quad SU(2)
\end{aligned}$$ (9)

$$\begin{aligned}
&\simeq [-8.4102 - 0.50203f + 0.10845f^2 - 2.066 \times 10^{-3}f^3 \\
&\quad -6.78 \times 10^{-5}f^4] \cdot [1 - 2f/33]^{-2} \quad \text{for} \quad SU(3),
\end{aligned}$$ (10)

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where $d^{abc}d^{abc} = 0$ for $SU(2)$ and $40/3$ for $SU(3)$ have been used. Then I investigate whether Eq. (2) has a positive solution if $f > 12\,(17)$ for $SU(2)\,(SU(3))$. The result is shown in TABLE I.

TABLE I. The third-order fixed points ($\alpha^*_{\mathrm{opt}} = a^*_{\mathrm{opt}}\pi$) from the R ratio.

	$SU(2)$			$SU(3)$	
f	$(b_0/b_1)\rho_2$	α^*_{opt}	f	$(b_0/b_1)\rho_2$	α^*_{opt}
12	-3.317	0.494	17	-18.197	0.096
13	-1.912	0.856	18	-5.689	0.294
14	-1.815	0.960	19	-3.794	0.441
15	-2.014	0.940	20	-3.365	0.516

As one can see from TABLE I, α^*_{opt} for some cases is small so that one may trust the results.

B. The Gross-Llewellyn Smith sum rule

This sum rule says that the first moment of the isospin singlet structure function for the hadronic matrix element which describes deep inelastic processes is six at the parton model level; $\int_0^1 dx\,(\,F_3^{\bar{\nu}p} + F_3^{\nu p}\,)(x, Q^2/\mu^2, a) = 6\,(\,1 + \mathcal{R}(Q^2/\mu^2, a)\,)$, where x is one of the scaling variables in the processes. The third-order QCD correction has been computed by Larin and Vermaseren [11]:

$$\mathcal{R}^{(3)} = \frac{3}{4}C_F\,a\,(\,1 + r_1 a + r_2 a^2\,)\,, \tag{11}$$

$$r_1(Q^2/\mu^2 = 1) = \frac{23}{12}C_A - \frac{7}{8}C_F - \frac{1}{3}f\,, \tag{12}$$

$$\begin{aligned}
r_2(Q^2/\mu^2 = 1) =\ & [\,\frac{5437}{648} - \frac{55}{18}\zeta(5)]\,C_A^2 - [\,\frac{1241}{432} - \frac{11}{9}\zeta(3)\,]\,C_A C_F + \frac{1}{32}C_F^2 \\
& + [\,(\,-\frac{3535}{1296} - \frac{1}{2}\zeta(3) + \frac{5}{9}\zeta(5)\,)\,C_A + (\,\frac{133}{864} + \frac{5}{18}\zeta(3)\,)\,C_F \\
& + (\,\frac{11}{144} - \frac{1}{6}\zeta(3)\,)\,\frac{d^{abc}d^{abc}}{C_F N_C}\,]\,f + \frac{115}{648}f^2\,. \tag{13}
\end{aligned}$$

As in the case A, I insert the r_1 and r_2 into the r.h.side of Eq. (3) and obtain

$$\begin{aligned}
\rho_2 \simeq\ & [\,6.8068 - 3.90512f + 0.57496f^2 - 3.157 \times 10^{-2}f^3 \\
& + 5.48 \times 10^{-4}f^4\,]\cdot[\,1 - f/11\,]^{-2}\ \ \text{for}\ \ SU(2) \tag{14}
\end{aligned}$$

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$$\simeq [16.5809 - 6.45245f + 0.630222f^2 - 2.2537 \times 10^{-2}f^3$$
$$+2.44 \times 10^{-4}f^4] \cdot [1 - 2f/33]^{-2} \quad \text{for} \quad SU(3) . \tag{15}$$

The values of ρ_2 and α^*_{opt} for some different $f(> 11C_A/2)$ are shown in TABLE II.

TABLE II. The third-order fixed points ($\alpha^*_{\text{opt}} = a^*_{\text{opt}}\pi$) from the Gross-Llewellyn Smith sum rule.

$SU(2)$			$SU(3)$		
f	$(b_0/b_1)\rho_2$	α^*_{opt}	f	$(b_0/b_1)\rho_2$	α^*_{opt}
12	-2.896	0.568	17	-17.196	0.100
13	-1.279	1.133	18	-4.681	0.339
14	-1.063	1.333	19	-2.766	0.558
15	-1.104	1.314	20	-2.296	0.684

The results are surprisingly similar to those for A. This again supports the reliability of the fixed point analysis based on OPT, and may be seen as an evidence for ultraviolet fixed points in the ANF Yang-Mills theories.

Triviality of gauged Higgs-Yukawa systems is widely expected, unless they are completely asymptotically free. A rigorous treatment of the asymptotic behavior of theory with more than one couplings is given in Ref. [12]. It was found [13] that by imposing a certain relation among the gauge, Higgs, and Yukawa couplings which are consistent with perturbative renormalizability, it is possible to make the $SU(3)$-gauged Higgs-Yukawa system completely asymptotically free and hence nontrivial [14]. This renormalization group invariant relation among couplings is a consequence of the "reduction of couplings" [12].

Inspired by the possibility that ANF Yang-Mills gauge theories may be nontrivial under certain circumstances and by the fact that gauged Higgs-Yukawa systems can be made asymptotically free by means of the reduction of couplings, one may be naturally led to the idea that even ANF gauged Higgs-Yukawa systems are nontrivial if the reduction of couplings is appropriately carried out. One then would achieve a *dynamical gauge-Higgs-Yukawa unification* in a theory, because these couplings are forced in a dynamically

consistent fashion to be related with each other in order for the theory to remain well-defined and interacting in the ultraviolet limit.

OPT for systems with more than one couplings does not exist yet, because there is no known systematic way how to control the propagation of the RS-dependence of lower orders to higher orders. But it is clear that once the reduction of couplings is applied to a system with many couplings so that the reduced system contains only one independent coupling, one can employ all the facilities of OPT. Unfortunately, third-order calculations in gauged Higgs-Yukawa systems do not exist yet. Here I would like to present the result of the two-loop reduction in the ANF $SU(3)$-gauged Higgs-Yukawa theory to motivate corresponding higher order calculations.

Let me first mention few words about the reduction of couplings, and consider a massless, renormalizable gauge theory based on a simple gauge group with N other couplings, where the gauge coupling is denoted by α, and the others by α_i , $i = 1, \cdots, N$. The complete reduction of couplings [12] is equivalent to demand that α_i be written as a power series of α, i.e., $\alpha_i = \sum_{n=0}^{\infty} \eta_i^{(n)} (\alpha/\pi)^n a$, $i = 1, \cdots, N$. As the consequence, the reduced system contains only α as the independent coupling–unification of couplings. It was shown [12] that the power series is consistent with perturbative renormalizability only if the reduction equations

$$\beta_\alpha(\alpha, \alpha_i(\alpha)) \frac{d\alpha_i(\alpha)}{d\alpha} = \beta_i(\alpha, \alpha_i(\alpha)) \tag{16}$$

are satisfied, where β_α stands for the β-function of α, and β_i for that of α_i. The uniqueness of the power series solution can be decided at the one-loop level, and the η's can be computed order by order in perturbation theory [12].

The gauged Higgs-Yukawa model I consider below can be obtained from the standard model by switching off the $SU(2)$ and $U(1)$ gauge couplings, dropping all leptons, and allowing n_d families of quarks. I also assume that only one of the (up-type) Yukawa couplings is nonvanishing; the simplified system contains only the $SU(3)$ gauge coupling α, the Yukawa coupling α_t, and the Higgs self-coupling α_h. (For $n_d \leq 8$, this system can be made completely asymptotically free [13].) Here I am interested in the case for $n_d > 8$,

and recall the β-functions [17]

$$\frac{\beta_3}{\pi} = a^2 \left[-\frac{11}{2} + \frac{2}{3}n_d + \left(\frac{19}{6}n_d - \frac{51}{4} \right)a - \frac{1}{4}a_t + \cdots \right] , \tag{17}$$

$$\frac{\beta_t}{\pi} = a_t \left[\frac{9}{4}a_t - 4a + \frac{9}{2}aa_t - \frac{3}{4}a_h a_t - \frac{3}{2}a_t^2 + \left(\frac{10}{9}n_d - \frac{101}{6} \right)a^2 + \frac{3}{16}a_h^2 + \cdots \right] , \tag{18}$$

$$\frac{\beta_h}{\pi} = 3a_h^2 + 3a_h a_t - 3a_t^2 - 4aa_t^2$$

$$- \frac{3}{16}a_h a_t^2 + \frac{15}{4}a_t^3 - \frac{39}{8}a_h^3 + 5aa_h a_t - \frac{9}{2}a_h^2 a_t + \cdots , \tag{19}$$

where $a_i = \alpha_i/\pi$. It can be shown that the power series solution of the reduction equations
(16) with $i = t$, h, i.e.,

$$\alpha_i = \sum_{n=0}^{\infty} \eta_i^{(n)} \left(\frac{\alpha}{\pi} \right)^n a , \ i = t, h ,$$

exists uniquely to all orders in perturbation theory so that the original system with three
independent couplings can uniquely be reduced to a system with only one independent
coupling, α. The first- and second-order coefficients can be computed by solving Eq. (16)
with the second-order β-function (17)-(19), and the results are given in TABLE III.

TABLE III. The expansion coefficients for the reduction of couplings in the

$SU(3)$-gauged Higgs-Yukawa theory.

n_d	$\eta_t^{(0)}$	$\eta_t^{(1)}/\pi$	$\eta_h^{(0)}$	$\eta_h^{(1)}/\pi$
9	2	3.294	1.283	2.586
10	2.296	4.356	1.533	3.592

The reduced system has only one β-function

$$\frac{\beta}{\pi} = a^2 \left[-\frac{11}{2} + \frac{2}{3}n_d + \left(-\frac{151}{12} + \frac{169}{54}n_d \right)a + O(a^2) \right] .$$

The fact that the first two coefficients of β for $n_d \geq 9$ are positive (as they are in the
previous cases) does not mean anything about a fixed point within the framework of OPT;
one needs a complete third-order calculation to obtain ρ_2 and then to solve Eq. (2). If it
will be negative and large, there will be a small, positive a_{opt}^*.

There will be many applications of the idea of dynamical unification of couplings
(DUC) in constructing realistic unified gauge models. Unification of the gauge couplings
in ANF extensions of the standard model, for instance, were previously considered in

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Refs. [15]. In contrast to the present idea, it was assumed there that the gauge couplings asymptotically diverge so that if one requires the couplings to become strong simultaneously at a certain energy scale, one can predict their low energy values [16]. There are many papers based on this idea, but non of them discusses nontriviality of ANF unified gauge models and its possible relation to unification of couplings. Obviously, it is desirable to justify the assumptions (specified in the text) leading to the idea of DUC independently in different approaches.

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